On the structure of truth definitions

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In the area of axiomatic theories of truth one studies formal properties of various consistent sets of meaning postulates (axioms) for the truth predicate. Importantly, in this approach, the truth predicate is treated as a primitive, much in the same way the membership relation is treated in contemporary set-theory. Each axiomatic theory of truth for a language $\mathcal{L}$ is a theory in the language extending $\mathcal{L}$ with a fresh predicate $T(x)$ and contains some axioms which jointly imply that T is a truth predicate for $\mathcal{L}$. From this perspective, a definition of truth for a first-order language $\mathcal{L}$ is any sentence σ from which all instantiations of the T-scheme for $\mathcal{L}$ can be derived. In other words: a definition of truth for $\mathcal{L}$ is a finite axiomatic theory of truth for $\mathcal{L}$. The main focus of my talk is the following question: given a first-order language $\mathcal{L}$, how different can the definitions of truth for $\mathcal{L}$ be?

One of the first design choices one has to make in order to start answering this question: when shall we treat two truth definitions as the same? One obvious candidate is: A and B are equivalent if A proves B and B proves A. We choose a more liberal path: we treat A and B as equivalent if A proves B modulo a translation of the truth predicate of B into the language if A and *vice versa*: B proves A modulo a translation of the truth predicate of A to the language of B. More precisely: we say that A *defines* B if there is a formula $\theta(x)$ such that A proves $B[\theta(x)/T(x)]$, where the latter denotes the result of substituting $\theta(x)$ for every occurrence of $T(x)$ in B. We treat A and B as equivalent if they are mutually definable. And we treat B as *stronger* than A if B defines A but not *vice versa*. The main goal of the talk is to explain some recent results about the structural properties of the relation of definability on truth definitions. In particular we shall show that many different patterns can be realized in this structure: we show that this structure is a distributive lattice which contains a copy of a countable and atomless Boolean algebra. As a consequence this structure is capable of embedding *any* countable and distributive lattice. Hence, in fact, there are many different truth definitions.

This is joint work with Piotr Gruza.

Introductory literature (links):

- Mateusz Łełyk, Bartosz Wcisło, Strong and Weak Truth Principles, Semiotic Studies, vol. 30, no. 2 (2016)
- Mateusz Łełyk, Comparing Axiomatic Theories of Truth, Semiotic Studies, vol. 33, no. 2, DOI: http://doi.org/10.26333/sts.xxxiii2.08, (2019)